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THE PERMISSIBLE RANGE OF THE RESULTS OF THE BÖHME ABRASION TESTS ON BUILDING STONES

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ABSTRACT

The abrasion resistance of building stones utilized to traffic exposed surfaces is generally controlled by the BÖHME test method. An increasing importance of this test due to the increasing frequency of stone-surfacing can be observed.

The standards regulating the execution of the Böhme test prescribe that the results acceptables have to be calculated as an average value of the single but compatible results. This paper deals with the condition of this compatibility.

The mesure of compatibility is the empirical dispersion of the test results or the empirical – but compatible – extent connected with the former, not exceeding a prescribed limit.

After analysing 318 results this extent acceptable was calculated, its comparison with the empirical extent gives us informations concerning the experimental error and the errors in the formation of specimengroupes.

I N T R O D U C T I O N

In Hungary the abrasion resistance of natural building and paving stones is determined according to the Böhme method (in compliance to the Hungarian standard MSZ 18290-1:1981). The authentic result is computed from the findings of the individual tests.

Authentic results must be calculated from measurements taken in the same laboratory with the same instruments and methods, by the same person, on specimens of the same group, viz. under reproducible conditions. A further criterion is that the results should be reliable and compatible (3).

However, neither the concept of compatibility nor its checking are specified in Hungarian or foreign Böhme standards, the compatibility of the tests being assumed. Its examination is important as it provides information about the reliability of the arithmetic mean calculated from the test results.

The measure of compatibility the ratio of the empirical scatter of the test findings, or the differences between the greatest and smallest results, viz. the ratio of the empirical range of the permissible value.

What follows is a method based on experiments proposed for the examination of the ratio of the empirical to the permissible range of the results of the Böhme abrasion test.

MATHEMATICAL, STATISTICAL BASES OF COMPATIBILITY

The empirical range has been chosen for the measure of the fluctuations of the measured values because it is easy to compute and, although its random fluctuations are greater than those of the empirical scatter, it helps to estimate the theoratical scatter. By reducing the empirical range R to a value below the permissible range R_m we reduce the empirical and the theoretical scatters and σ of the measurement results, as at a given statistical probability of P_R the empirical range R is closely related to the theoretical scatter σ of the measured values. Assuming the normal distribution of the measured values this relationship is as foltlows:

$$R = \omega \cdot \sigma$$

which means that the empirical range is the product of the threshold value ω and the theoretical scatter, so, to establish the permissible range R_m the same threshold value must be multiplied by the permiseible scatter s_m:

$$R_m = \omega \cdot s_m$$

The product $\omega \cdot s$ is that interval ofthe abscissa on one side of the ofthe curve, into which the empirical range R, as a probability variable, at a given probability or P_R, is likely to fall. The probability P_R is described by the density function ot the standard probability variable R/σ (4,5), its related values P_R and ω, as a function ofthe number ofmeasurement n are compiled in tables (1, 2). The threshold value ω accordingly depends on the statistical probability P_R and the number of measurements n:

$$\omega = f(P_R \cdot n)$$

The value of the statistical probability P_R on which the threshold value, ω depends is a matter of convention. With a given permissible scatter a greater P_R value represents a higher threshold value ω and through it a broader range of R_m, less rigorous criteria.

To compute the authentic result both the Hungarian add foreign practices apply a 95% statistical probability related to the 2ω interval of the measured values. In our tests we followed the same practice. In it the empirical range will transgress the permissible value only in five cases from 100. The threshold values pertaining to the statistical probability P_R=0,95 with three to six measurements were as follows (1,2):

n	ω
3	3.314493
4	3.633160
5	3.857656
6	4.030092

The permissible range is to be determined from these threshold values and the empirical scatter ofa large number of measurements carried out with care, which are regarded to represent the permissible scatter.

COMPUTING THE PERMISSIBLE RANGE

The Böhme abrasion test which forms the basis of the Hungarian standard MSZ 18290-1:1981 is fully acceptable for the determination of the permissible range. Three hundred and eighteen measurements were taken which yielded 106 average values. During the tests cubes and cylinders were abraded by KA 10-12 and KA 32 quality electrocorundum and natural Naxos corundum, according to both the dry and the wet methods, by 352 revolutions. The wear values were assumed with the correcion factors of the Hungarian standard and related to the wear of sepcimen cubes with 7,1 cm edge length abraded with KA 10-12 electrocorundum.

The standard deviation and the range increase with increasing average value. This proves that it is erroneous to determine the permissible range independently from the average of the results. Accordingly, the permissible range is not a single datum but a function, so the permissible range schould be computed with the standard deviation as the function of the average value, and not from the scatter of the anyway unrelated set of test findings.

The regressive relationship between the average value and the standard deviation was expressed by the second-degree central parabola

$$y = \mathrm{a}x^2 + \mathrm{b}x$$

where y denotes the standard deviation, x the variable of the average value. The coefficients ofthis relationship were determined on the least sum of squared errors, separately for the loss of volume and the loss of height. The values of the relative error of the coefficients of „a" and „b", the correlation index I, the absolute error of S and the relative error of H can be found in *Table 1*.

Accordingly, the permissible range must be computed as the product of the threshold value and the permissible scatter. Let the permissible scatter be equal to the theoretical scatter σ, where the distorted estimation is represented by the empirical scatter s.

It has been verifled that the undistorted estimation of the empirical scatter corrected witb the multiplier

$$\sqrt{\frac{n}{n-1}}$$

is the undistorted estimation of the theoretical scatter (5). In our tests n = 3. So to compute the corrected empirical.scatter the non-corrected scatter must be multiplied by 1.225 viz. the standard deviation.

Since the corrected empirical scatter is 1.225 times the standard deviation n, also the coefficients of the relationship betwwen the average value and the corrected empirical scatter is 1.225 times the coefficients in *Table 1*. These corrected coeffifients are characteristic of the relationship between the average value and the permissible scatter, viz. $y = ax^2 + bx$. Theirr values are indicated in *Table 2*, their curves are illustrated in *Fig.1*.

The permissible range related to some average value of the Böhme abrasion tests can be computed by multiplying the permissible scatter s_m determined by the coefficients of *Table 2* with a threshold value ω, corresponding to the number of tests taken.

Making use of the coefficients of *Table 2* we computed the permissible scatters. Having multiplied them by the threshold value of ω_3 we obtained the permissible ranges. We established that the range of our test results transgressed the permissible range eight times on the loss of volume, ten times on the loss of height and six times on both. Having accepted this latter as the authentic value, from the altogether 106 groups the compatibility criterion was satisfied in 100 cases, viz. in 94.3% - very close to the 95% statistical probability according to the convention. This verifies the applicability and correctness of our method.

EXAMINATION OF THE COMPATIBILITY

To facilitate the examination of compatibility, the permissible ranges, in function of the average values and the mumber of tests, were compiled into tables. *Table 3* shows the permissible scatter s_m related to the 1.0 cm^3 accuracy average values of losses and the R_{m3-m6} permissible ranges obtained in three to six measurements. *Table 4* shows the same particulars related to the average loss of height at an accuracy of 0.2 mm. The permissible ranges, in function of the average values, were graphically represented in *Figures 2* and *3*.

To establish the compatibility of the test results so as to compute the authentic results of the Böhme abrasion test, the computed average values must be rounded to 1.0 cm^3 and 0.2 mm, then, in consideration of the number of tests, the permissible ranges for the average value should be read

from *Tables 3* and *4* and compared to the empirical ranges. If at least one. of the two empirical ranges (loss of volume and height) is below the permissible, then the average can be accepted as compatible, and the test results are authentic.

In the contrary case on the other hand – provided that the test results are faultless – the tests must be repeated as many times as the increased number of tests, on new specimens from the same group. If even this method fails to yield authentic results, the results farthest away from the average should be omitted, and a check performed on the remaining results – which is the original number. This procedure should be continued until a compatible value is obtained at least for one of the two losses. If this process still fails to verify the compatibility then no authentic result should be derived from the test results. The authentic loss of mass and weight must naturally be computed from tests performed on the same specimens.

As regards the above method for the computation of an authentic result we may state that the comparison of the actual with the permissible range of the test results may reveal errors not only in the tests but also in the specimen group. As such, the method may avert the consequences of both errors.

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TABLE 1.

COEFFICIENTS AND CORRELATION CHARACTERISTICS OF THE RELATIONSHIP BETWEEN THE AVERAGE VALUE AND THE STANDARD DEVIATION

Parameter	Loss of volume	Loss of height
a	- 0.000302652286	- 0.003159902718
b	+ 0.066319533	+ 0.077485345
I	0.5982	0.5733
S	0.8683 cm^3	0.1741 mm
H	78.30 %	75.61 %

TABLE 2.

COEFFICIENTS OF THE RELATIONSHIP BETWEEN THE AVERAGE VALUE AND THE PERMISSIBLE SCATTER

Parameter	Loss of volume	Loss of height
a	- 0.000370749050	- 0.003870880830
b	+ 0.081241428	+ 0.094919548

TABLE 3.

THE PERMISSIBLE SCATTER AND THE RANGE OF THE LOSS OF VOLUME IN BÖHME ABRASION TESTS (cm^3)

Average value	s_m	R_{m3}	R_{m4}	R_{m5}	R_{m6}
1	0.0809	0.27	0.29	0.31	0.33
2	0.1610	0.53	0.58	0.62	0.65
3	0.2404	0.80	0.87	0.93	0.97
4	0.3190	1.06	1.16	1.23	1.29
5	0.3969	1.32	1.44	1.53	1.60
6	0.4741	1.57	1.72	1.83	1.91
7	0.5505	1.82	2.00	2.12	2.22
8	0.6262	2.08	2.28	2.42	2.52
9	0.7011	2.32	2.55	2.70	2.83
10	0.7753	2.57	2.82	2.99	3.12
11	0.8488	2.81	3.08	3.27	3.42
12	0.9215	3.05	3.35	3.55	3.71
13	0.9935	3.29	3.61	3.83	4.00
14	1.0647	3.53	3.87	4.11	4.29
15	1.1352	3.76	4.12	4.38	4.57
16	1.2050	3.99	4.38	4.65	4.86
17	1.2740	4.22	4.63	4.91	5.13
18	1.3422	4.45	4.88	5.18	5.41
19	1.4097	4.67	5.12	5.44	5.68
20	1.4765	4.89	5.36	5.70	5.95
21	1.5426	5.11	5.60	5.95	6.22
22	1.6079	5.33	5.84	6.20	6.48
23	1.6724	5.54	6.08	6.45	6.74
24	1.7362	5.75	6.31	6.70	7.00
25	1.7993	5.96	6.54	6.94	7.25

Average value	s_m	R_{m3}	R_{m4}	R_{m5}	R_{m6}
26	1.8617	6.17	6.76	7.18	7.50
27	1.9232	6.37	6.99	7.42	7.75
28	1.9841	6.58	7.21	7.65	8.00
29	2.0442	6.78	7.43	7.89	8.24
30	2.1036	6.97	7.64	8.11	8.48
31	2.1622	7.17	7.86	8.34	8.71
32	2.2201	7.36	8.07	8.56	8.95
33	2.2772	7.55	8.27	8.78	9.18
34	2.3336	7.73	8.48	9.00	9.40
35	2.3893	7.92	8.68	9.22	9.63
36	2.4442	8.10	8.88	9.43	9.85
37	2.4984	8.28	9.08	9.64	10.07
38	2.5518	8.46	9.27	9.84	10.28
39	2.6045	8.63	9.46	10.05	10.50
40	2.6565	8.80	9.65	10.25	10.71
41	2.7077	8.97	9.84	10.45	10.91
42	2.7581	9.14	10.02	10.64	11.12
43	2.8079	9.31	10.20	10.83	11.32
44	2.8569	9.47	10.38	11.02	11.51
45	2.9051	9.63	10.55	11.21	11.71
46	2.9526	9.79	10.73	11.39	11.90
47	2.9994	9.94	10.90	11.57	12.09
48	3.0454	10.09	11.06	11.75	12.27
49	3.0907	10.24	11.23	11.92	12.46
50	3.1352	10.39	11.39	12.09	12.64

TABLE 4.

THE PERMISSIBLE SCATTER AND THE RANGE OF THE LOSS OF HEIGHT IN BÖHME ABRASION TESTS (mm)

Average value	s_m	R_{m3}	R_{m4}	R_{m5}	R_{m6}
0.2	0.0188	0.06	0.07	0.07	0.08
0.4	0.0373	0.12	0.14	0.14	0.15
0.6	0.0556	0.18	0.20	0.21	0.22
0.8	0.0735	0.24	0.27	0.28	0.30
1.0	0.0910	0.30	0.33	0.35	0.37
1.2	0.1083	0.36	0.39	0.42	0.44
1.4	0.1253	0.42	0.46	0.48	0.50
1.6	0.1420	0.47	0.52	0.55	0.57
1.8	0.1583	0.52	0.56	0.59	0.62
2.0	0.1744	0.58	0.63	0.67	0.70
2.2	0.1901	0.63	0.69	0.73	0.77
2.4	0.2055	0.68	0.75	0.79	0.83
2.6	0.2206	0.73	0.80	0.85	0.89
2.8	0.2354	0.78	0.86	0.91	0.95
3.0	0.2499	0.83	0.91	0.96	1.01
3.2	0.2641	0.88	0.96	1.02	1.06
3.4	0.2780	0.92	1.01	1.07	1.12
3.6	0.2915	0.97	1.06	1.12	1.17
3.8	0.3048	1.01	1.11	1.18	1.23
4.0	0.3177	1.05	1.15	1.23	1.28
4.2	0.3304	1.10	1.20	1.27	1.33
4.4	0.3427	1.14	1.25	1.32	1.38
4.6	0.3547	1.18	1.29	1.37	1.43
4.8	0.3664	1.21	1.33	1.41	1.48
5.0	0.3778	1.25	1.37	1.46	1.52

Average value	s_m	R_{m3}	R_{m4}	R_{m5}	R_{m6}
5.2	0.3889	1.29	1.41	1.50	1.57
5.4	0.3997	1.32	1.45	1.54	1.61
5.6	0.4102	1.36	1.49	1.58	1.65
5.8	0.4203	1.39	1.53	1.62	1.69
6.0	0.4302	1.43	1.56	1.66	1.72
6.2	0.4397	1.46	1.60	1.70	1.77
6.4	0.4489	1.49	1.63	1.73	1.81
6.6	0.4579	1.52	1.66	1.77	1.85
6.8	0.4665	1.55	1.69	1.80	1.88
7.0	0.4748	1.57	1.72	1.83	1.91
7.2	0.4828	1.60	1.75	1.86	1.95
7.4	0.4904	1.63	1.78	1.89	1.98
7.6	0.4978	1.65	1.81	1.92	2.01
7.8	0.5049	1.67	1.83	1.95	2.03
8.0	0.5116	1.70	1.86	1.97	2.06
8.2	0.5181	1.72	1.88	2.00	2.09
8.4	0.5242	1.74	1.90	2.02	2.11
8.6	0.5300	1.76	1.93	2.04	2.14
8.8	0.5355	1.78	1.95	2.07	2.16
9.0	0.5407	1.79	1.96	2.09	2.18
9.2	0.5456	1.81	1.98	2.10	2.20
9.4	0.5502	1.82	2.00	2.12	2.22
9.6	0.5545	1.84	2.01	2.14	2.23
9.8	0.5585	1.85	2.03	2.15	2.25
10.0	0.5621	1.86	2.04	2.17	2.27

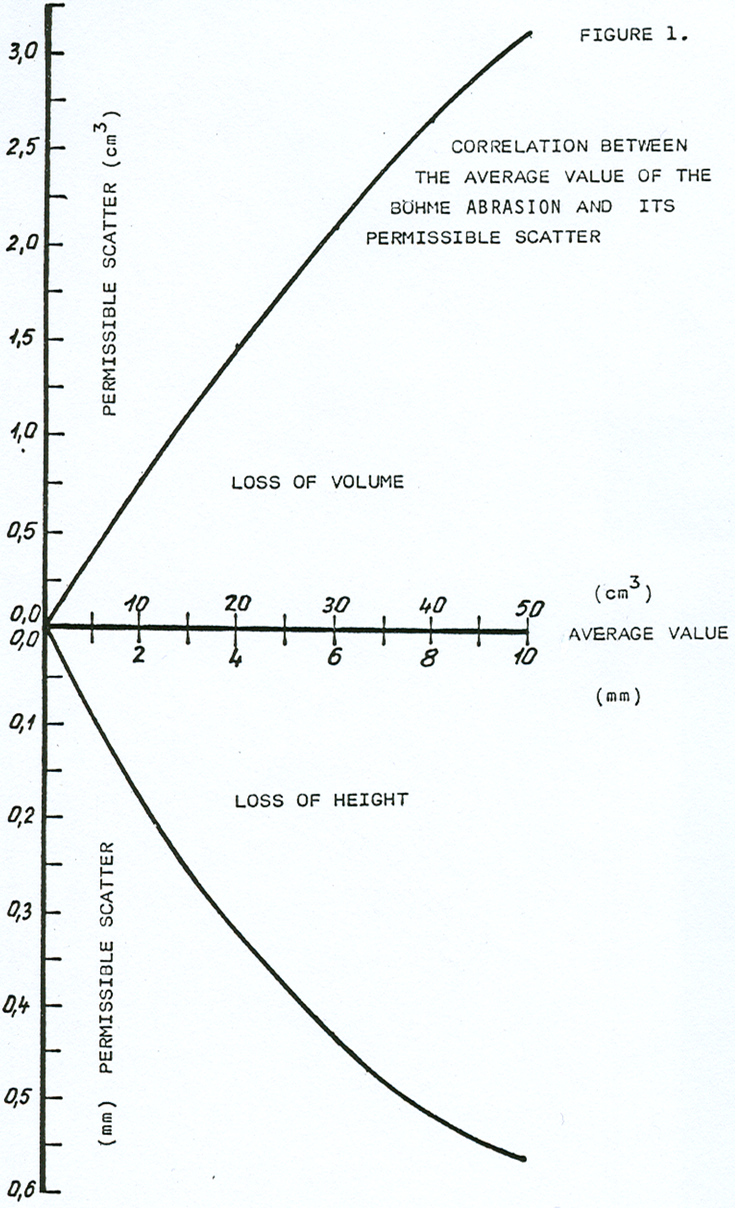
FIGURE 1.
CORRELATION BETWEEN THE AVERAGE VALUE OF THE BÖHME ABRASION AND ITS PERMISSIBLE SCATTER
PERMISSIBLE SCATTER (cm^3)
3,0
2,5
2,0
1,5
1,0
0,5
0,0
0,0
0,1
0,2
0,3
0,4
0,5
0,6
LOSS OF VOLUME
10
20
30
40
50
(cm^3)
AVERAGE VALUE
2
4
6
8
10
(mm)
LOSS OF HEIGHT
(mm) PERMISSIBLE SCATTER

Figure 2.

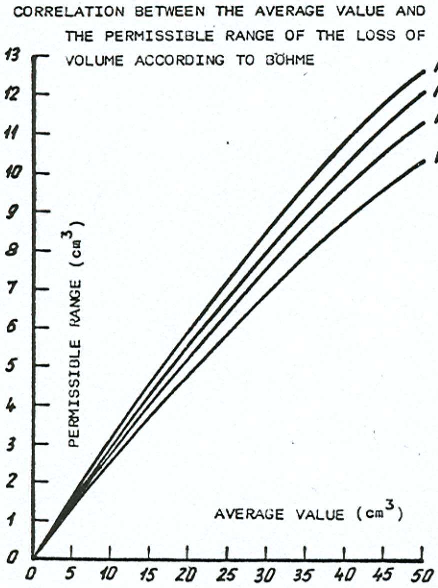

FIGURE 3.